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EXACT RENORMALIZATION GROUP AND MANY-FERMION SYSTEMS

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The exact renormalization group method is applied to many-fermion systems with short-range attractive forces. The strength of the attractive fermion-fermion interaction is determined from the vacuum scattering length. A set of approximate flow equations is derived including fermionic bosonic fluctations. The numerical solutions show a phase transition to a gapped phase. The inclusion of bosonic fluctuations is found to be significant only in the small-gap regime.

Keywords: Superfluidity; Pairing, Renormalization Group; Phase Transition.

Attractive forces between fermions play a crucial role in many areas of many-body physics. In a system of fermions, any attraction can cause such particles to form correlated "Cooper pairs" so that a phase transition occurs. The order parameter for this is the energy gap, which appears in the fermion spectrum. Within such systems we can identify a weak-coupling regime, with no two-body bound states, which corresponds to Bardeen-Cooper-Schrieffer (BCS) superconductivity and a strong-coupling regime, with a deeply bound two-body state, which manifests itself in a form of Bose-Einstein Condensation (BEC).

A new and promising way to treat many-fermion systems with these different regimes is provided by the Exact Renormalization Group (ERG)[1]. This has been successfully applied to a variety of other systems in particle and condensed-matter physics[2]. The goal of the ERG approach is to construct the Legendre transform of the effective action, Γ, which generates the 1PI Green's functions. One introduces an artificial renormalisation group flow for Γ, depending on a momentum scale k, and defines a running effective action by integrating over components of the fields with $q < k$. The RG trajectory then interpolates between the classical action of the underlying theory (at large k), and the full effective action (at $k = 0$)[2].

Here we study a system of fermions interacting through an attractive two-body contact potential. We take as our starting point an EFT that describes the s-wave interaction of two fermions with scattering length a_0. Since we are interested in the appearance of a gap in the fermion spectrum, we introduce a boson field whose vacuum expectation value (VEV) describes that gap and so acts as the corresponding

order parameter. We take the following Ansatz for Γ:

$$\Gamma[\psi,\psi^\dagger,\phi,\phi^\dagger,\mu,k]=\int d^4x\,\left[\phi^\dagger(x)\left(Z_\phi\, i\partial_t+\frac{Z_m}{2m}\nabla^2\right)\phi(x)-U(\phi,\phi^\dagger)\right.$$
$$\left.+\psi^\dagger\left(Z_\psi(i\partial_t+\mu)+\frac{Z_M}{2M}\nabla^2\right)\psi-Z_g\left(\frac{i}{2}\,\psi^{\mathrm{T}}\sigma_2\psi\phi^\dagger-\frac{i}{2}\,\psi^\dagger\sigma_2\psi^{\dagger\mathrm{T}}\phi\right)\right].\quad(1)$$

Here M is the mass of the fermions in vacuum and m is taken to be $2M$.

We expand the potential U about its minimum, $\phi^\dagger=\phi=\Delta$ where Δ is the fermion energy gap. To quartic order this has the form

$$U(\phi,\phi^\dagger)=u_0+u_1(\phi^\dagger\phi-\Delta^2)+\tfrac{1}{2}\,u_2(\phi^\dagger\phi-\Delta^2)^2.\quad(2)$$

In the RG evolution, we start at high k from an almost free bosonic action where the VEV of ϕ is zero. When k is lowered, we expect that at some scale the system undergoes a transition to a phase with spontaneously broken $U(1)$ symmetry and a nonzero fermion gap Δ.

The evolution equation for Γ in the ERG has a straightforward one-loop structure[2] and in the case of constant μ it has the form

$$\partial_k\Gamma=-\frac{i}{2}\,\mathrm{Tr}\left[(\Gamma^{(2)}_{BB}-R_B)^{-1}\,\partial_kR_B\right]+\frac{i}{2}\,\mathrm{Tr}\left[(\Gamma^{(2)}_{FF}-R_F)^{-1}\,\partial_kR_F\right].\quad(3)$$

Here $\Gamma^{(2)}_{FF(BB)}$ is the matrix of the second derivatives of the effective action with respect to the fermion (boson) fields and $R_{B(F)}$ is a matrix containing the corresponding boson (fermion) regulators. The usual mean-field results can be recovered if we include fermion loops only (the second term of this equation). By evaluating the loop integrals in (3) we get a set of coupled equations for the coefficents u_n and Z_ϕ at constant μ. Details can be found in our paper[3]. The initial conditions are obtained by assuming that in vacuum our theory reproduces the scattering length a_0, and that the necessary subtractions are identical in matter and in vacuum.

Although our results can be applied to many systems, for definiteness we concentrate on parameter values relevant to neutron matter: $M=4.76$ fm^{-1}, $p_F=1.37$ fm^{-1}, and large two-body scattering lengths ($|a_0|>1$ fm). We have checked the dependence of our results on the starting scale K and find that this is undetectable as long as $K>5$ fm^{-1}. Some typical solutions for the evolution equations are given in Fig. 1, for the case of infinite a_0. We compare two different approximation schemes, one where we keep fermion loops only, and one where we include boson loops as well and we allow Z_ϕ to run. For large values of k the system remains in the symmetric phase. At $k\simeq 1.2$ fm^{-1}, the first derivative of U at $\phi=0$ vanishes and the system enters the broken phase.

We find that the contributions of boson loops are small. Indeed in the symmetric phase they have essentially no effect. Below the transition they do become visible, particularly in u_2. However their effects on the gap are even smaller, at most $\sim 1\%$ for $1/|p_Fa_0|<1$. In this region, the main effect of the bosons is a small enhancement of the gap related to the reduction in u_2.

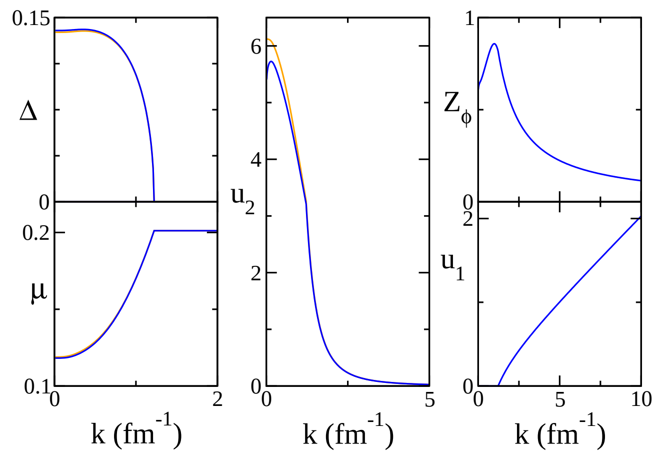

Fig. 1. Numerical solutions to the evolution equations for the relevant parameters for infinite a_0 and $p_F = 1.37$ fm, starting from $K = 16$ fm^{-1}. The black line shows the full evolution with both fermion and boson loops; the grey line the result with fermion loops only. All quantities are expressed in appropriate powers of fm^{-1}.

We have also examined the behaviour of the gap over a wider range of $1/(p_F a_0)$. The overall picture is the same as in Marani et al.[4] for fermion loops only, with a crossover from BCS pairing (with $\mu \simeq \epsilon_F$) for $1/(p_F a_0) < 0$ to BEC (with $\mu < 0$) for $1/(p_F a_0) > 0$. Deviations from mean-field behaviour are present in the BEC region and become increasingly noticable for weaker couplings or lower densities.

In the case of neutron matter, we find gaps of the order of 30 MeV. There is a simple explanation for such unrealistically large values[5]. For weak couplings, the gap is proportional to $\exp\left(-(\pi/2)\cot\delta(p_F)\right)$. For nucleon-nucleon scattering, $\cot\delta$ increases quickly with momentum, resulting in a reduction of the gap. We therefore expect that inclusion of the effective range should capture this physics.

There a number of improvements which should be made to our approach. Adding an effective range is clearly an important one. Notes that this will also require inclusion of the three-body forces to respect the reparametrization invariance[6]. The running of the boson kinetic mass (Z_m), the fermion renormalization constants and the "Yukawa" coupling are needed for a full treatment of the action (1). Beyond that we would also like to treat explicitly particle-hole channels (RPA phonons).

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